TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 3 2020

Section 1 Calculator Free

Discrete Random Variables

STUDENT'S NAME SOLUTIONS [KRISZYK]

DATE: Thursday 14th May **TIME:** 15 minutes **MARKS:** 18

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

(a) Determine if each of the following are probability functions, justify your answer either way.

(i) [2]

x	-1	1	2	3
$P(X=x)$	0	0.3	0.4	0.3

Yes ✓

$P(X=x) \geq 0$ for all x

$\sum P(X=x) = 1$ ✓

(ii) $P(X=x)=\dfrac{x^2}{30}$; $x=1,2,3,4$ [3]

x	1	2	3	4
$P(X=x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

✓

Yes ✓

$P(X=x) \geq 0$ ✓

$\sum P(X=x) = 1$

(b) An experiment is conducted where a ball is randomly picked from a bag containing red and green balls. What condition(s) must be placed on the experiment for it to be considered a Bernoulli experiment? [2]

- Sucess / Fail criteria must be defined.
- one trial

2. (11 marks)

The discrete random variable X has a cumulative probability distribution given by:

x	-1	0	1	2	3
$P(X \leq x)$	0.2	$0.2 + a$	$0.3 + a$	$0.3 + 2a$	$0.5 + 2a$
	0.2	0.45	0.55	0.8	1

x	-1	0	1	2	3
$P(X=x)$	0.2	0.25	0.1	0.25	0.2

(a) Determine a using the distribution above. [2]

$2a + 0.5 = 1$

$a = 0.25$

$\frac{13}{60}$ if $a = \frac{-1}{12}$

(b) Calculate:

(i) $P(X < 2)$ [1]

0.55

(ii) $P(X > -1 \mid X \leq 2)$ [2]

$\frac{0.6}{0.8} = 0.75$

$\cancel{\frac{7}{15}}$

impossible if $a = \frac{-1}{12}$

(b) Determine E(X) [2]

$E(X) = (-1 \times 0.2) + (0 \times 0.25) + (1 \times 0.1) + (2 \times 0.25) + (3 \times 0.2)$

$= -0.2 + 0.1 + 0.5 + 0.6$

$= 1$

4.95 F/T if used cumulative

$\frac{77}{60}$ if $a = \frac{-1}{12}$ and cumulative

(c) If $E(X^2) = \frac{31}{10}$, determine Var (X). [2]

$Var(X) = \frac{31}{10} - (1)^2$

$= \frac{21}{10}$

The random variable $Y = 6 - 2X$

(d) (i) Determine $E(Y)$ [1]

$E(Y) = 6 - 2(1)$

$= 4$

(ii) Determine Var (Y) [1]

$Var(Y) = (-2)^2 \times \frac{21}{10}$

$= \frac{84}{10}$

TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 3 2020

Section 2 Calculator Assumed
Discrete Random Variables

STUDENT'S NAME SOLUTIONS [KRISZYK]

DATE: Thursday 14th May **TIME:** 35 minutes **MARKS:** 38

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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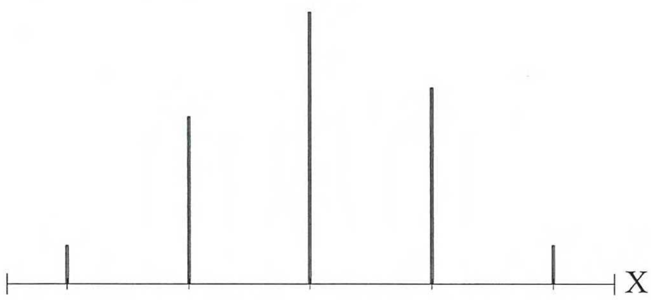
4. (3 marks)

This graph represents a binomial probability distribution.

The height of the last column is 0.053



(a) State the value of n. [1]

$n=4$

(b) State the probability of success for this binomial distribution, correct to 2 decimal places. [2]

$^4C_4 \times p^4 \times (1-p)^0 = 0.053$

$p = 0.4798$

$p = 0.48$

5. (8 marks)

In Australia, approximately 9% of people have the blood type O-negative. On any given day the Red Cross needs to collect blood from at least 2 donors with O-negative blood.

On Wednesday they collect blood from 20 donors of various blood types, let the random variable X be the number of donors with O-negative blood.

(a) State the probability distribution for X. [1]

$X \sim B(20, 0.09)$

(b) Calculate the probability that the target of at least 2 donors with O-negative blood will be met. [2]

$P(X \geq 2) = 0.5484$

(c) Given that at least 15 donors without O-negative blood have been found from the 20, what is the probability that there will be exactly 4 donors with O-negative blood? [3]

$P(X=4 \mid X \leq 5) = \dfrac{0.0702952}{0.9932105}$

$= 0.0708$

(d) How many donors would be needed to give at least a 99% chance that at least 1 donor with blood type O-negative is found? [2]

$P(X \geq 1) \geq 0.99$

$\Rightarrow P(X < 1) \leq 0.01$

$\Rightarrow P(X = 0) \leq 0.01$

$n = 49$

6. (4 marks)

A school has analysed the examination scores for all its Year 12 students taking Methods as a subject. Let X : the examination percentage scores of all the Methods Year 12 students at the school. The school found that the mean was 75 with a standard deviation of 22.

The school has decided to scale the results using the transformation $Y = aX + b$ where a and b are constants and Y : the scaled percentage scores. The aim is to change the mean to 60 and the standard deviation to 15.

Determine the values of a and b.

$15 = 22a$

$a = \frac{15}{22}$ ✓✓

$a \approx 0.682$

$60 = 75a + b$

$b = \frac{195}{22}$ ✓✓

$b \approx 8.864$

7. (8 marks)

Amal noticed that in Methods, which he attends four days a week, the chance of being set homework on any one day was 70% and independent of the previous day.

(a) Determine the probability that Amal is set homework in this class;

(i) the next time he attends but not the lesson after [1]

$0.7 \times 0.3 = 0.21$ ✓

(ii) exactly twice in the next week [2]

$X \sim B(4, 0.7)$ ✓ $= 0.2646$ ✓

(b) Determine the probability that over a ten-week term, Amal will be given homework twice a week in Methods, at least 7 times. [2]

$Y \sim B(10, 0.2646)$ ✓

$P(Y \geq 7) = 0.005$ ✓

Allow FT

(c) Determine that probability that Amal would receive his fourth lot of homework on his seventh lesson of Methods. [3]

Z : Amal gets homework in 6 lessons

$Z \sim B(6, 0.7)$ $P(Z=3) = 0.1852$ ✓

$\therefore P(\text{Fourth homework on } 7^{th}) = 0.1852 \times 0.7$ ✓

$= 0.1296$ ✓

8. (5 marks)

A discrete random variable X has the following probability distribution:

x	0	1	2	3
$P(X=x)$	0.064	0.288	y	z

Note: for all other x, $P(X=x)=0$

(a) Determine the value of y in terms of z. [1]

$y = 0.648 - z$

(b) Given X is a binomial random variable, determine the values of y and z. [4]

$X \sim B(3, p)$

$P(X=0) = {}^3C_0\, p^0 (1-p)^3$ ✓

$0.064 = (1-p)^3$

$p = 0.6$ ✓

$P(X=2) = {}^3C_2\, (0.6)^2 (0.4)$

$= 0.432$

$y = 0.432$ ✓

$\therefore z = 0.216$ ✓

9. (10 marks)

For Edmund Rice Day, one PCG will have a game involving rolling two regular six-sided dice. The **difference** in the values on the two dice will be the score. The game will cost $2 for an attempt with a payout back to the customer of $8 if the score is 5, a payout of $5 if the score is 2 and they get their money back if the score is 4. All other scores do not win a prize

(a) Indicate the possible outcomes for one attempt of rolling two dice. [2]

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Let X be the **profit** made by the PCG on one attempt.

(b) Complete the table below [2]

	Lose	$5 win	$8 win	MB
x	2	-3	-6	0
$P(X = x)$	$22/36$	$8/36$	$2/36$	$4/36$

The PCG is anticipating that there will be 600 attempts of the game on the day.

(c) How much total profit is the PCG expecting? [2]

$$E(X) = \frac{2}{9}$$

$$\text{Expected profit} = 600 \times \frac{2}{9}$$

$$= \$133.33$$

The PCG decide this is not enough profit and decides to increase the cost of one attempt so that the expected profit on the 600 rolls would be at least $400.

(d) Noting that all transactions are made with cash, what is the **minimum** charge to **ensure** expected profit is at least $400? [4]

	lose	\$5 win	\$8 win	MB
x	c	$c-5$	$c-8$	0
$P(X=x)$	$22/36$	$8/36$	$2/36$	$4/36$

✓

$$E(X) = \frac{22c}{36} + \frac{8(c-5)}{36} + \frac{2(c-8)}{36} + 0$$

✓✓

$$= \frac{32c-56}{36}$$

Expected total profit =

$$400 = 600 \times \frac{32c-56}{36}$$

$$c = \$2.50$$ ✓

i.e charge $2.50 or more.